Coordinate free concept of multiscale systems

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Multi scale models with an explicit subdivision to fast and slow subsystems and an explicit small parameter the main subject of the theory of singularly perturbed differential equations (SPS). Using the well developed method of fast and slow integral (invariant) manifolds such systems can be decomposed to low dimensional models that treat separately fast and slow motions (fast and slow subprocesses. The main obstacle with practical application of this approach is an implicit nature of the multi scale structure for complex dynamical models.

We will discuss a coordinate free concept of multiscale systems, so-called singularly perturbed vector fields (SPVF). Our definition of singularly perturbed vector fields is associated with a well known in geometry concept of vector bundles. Asymptotically the SPVF theory answers the following questions: existence of multi scale structure, evaluation of fast manifolds, evaluation of small parameter, analysis of slow integral (invariant) manifolds. A practical (numerical) implementation of the SPVF concept is more restricted. We are looking for a global change of coordinate that transform an original model to a standard singularly perturbed system. We call such procedure as a global quasi-linearization procedure (GQL). The main purpose of the GQL is a transformation of the fast submanifolds to coordinate subspaces that corresponds geometrically to standard SPS systems.

The GQL represents a robust automatic reduction procedure. First, we estimate a possible dimension reduction. Second, any standard reduction procedure can be applied at this step. If such procedures do not reduce an original model until an estimate possible dimension, we will use a coordinate transformation by the GQL procedure to reduce the original system until the estimated dimension. Third, any standard procedure of the SPS theory can be applied after this transformation.

Because the GQL procedure is not unique we will discuss a number of geometrical invariants for an optimal choice of the GQL procedure. Using a number of test examples and practical combustion models it has been shown that this approach can be successfully applied numerically. The robust automatic algorithm based on the GQL procedure, proposed in our previous papers has been modified for practical kinetic and combustion models in dimensional form.

References

Bykov V, Goldfarb I. Gol'dshtein V. Singularly Perturbed Vector Fields. Journal of Physics: Conference Series: (2006), 52, 22-44

Bykov V., Gol'dshtein V., Maas U. Simple Reduction Technique based on Decomposition approach, Combustion Theory and Modelling. (2008), 12(2), 389-405.