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# **Application of Modelling Methods in Wind Turbine Engineering**

**Andrej Horvat, David Hartwanger**

**Intelligent Fluid Solutions Ltd**

**Free flow turbines and their efficiency**

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Leicester, UK

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# Contact information

## Dr. Andrej Horvat

Principal Engineer

Intelligent Fluid Solutions Ltd.

99 Milton Park, Abingdon, Oxfordshire, OX14 4RY, UK

Tel.: +44 (0)1235 84 15 05

Fax: +44 (0)1235 85 40 01

Mobile: +44 (0)79 72 17 27 00

Skype: a.horvat

E-mail: [andrej.horvat@intelligentfluidsolutions.co.uk](mailto:andrej.horvat@intelligentfluidsolutions.co.uk)

Web: [www.intelligentfluidsolutions.co.uk](http://www.intelligentfluidsolutions.co.uk)



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Production methods and planning

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How ?

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Product performance assessment  
Design optimisation  
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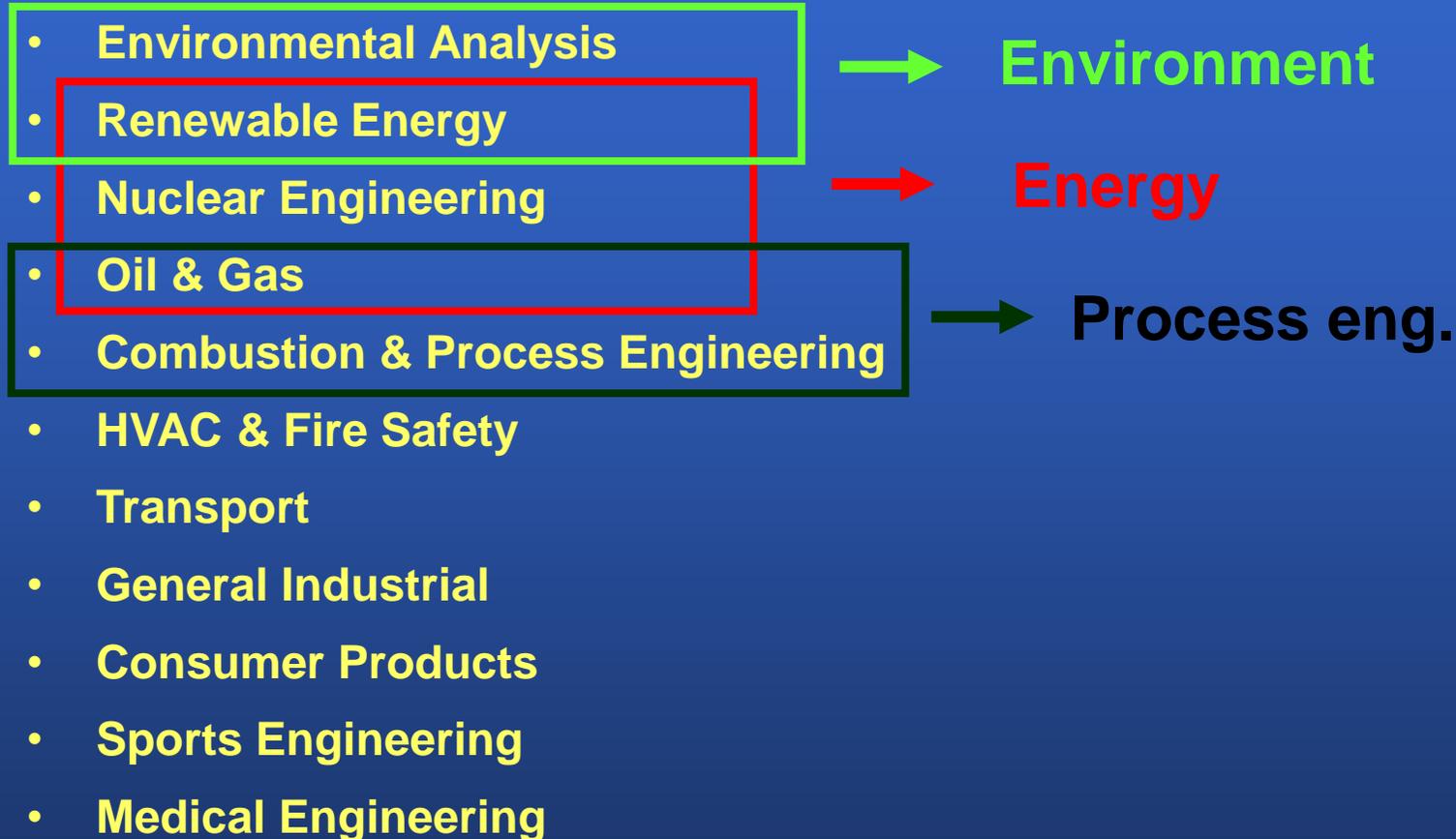
**Engineering  
analysis**

How ?

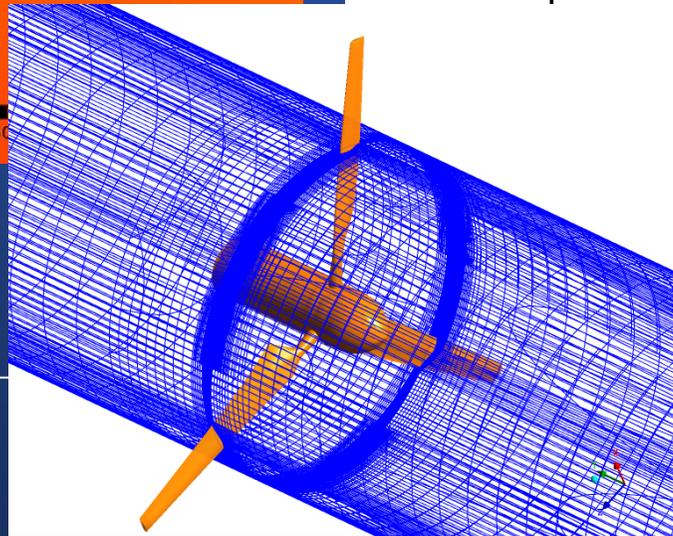
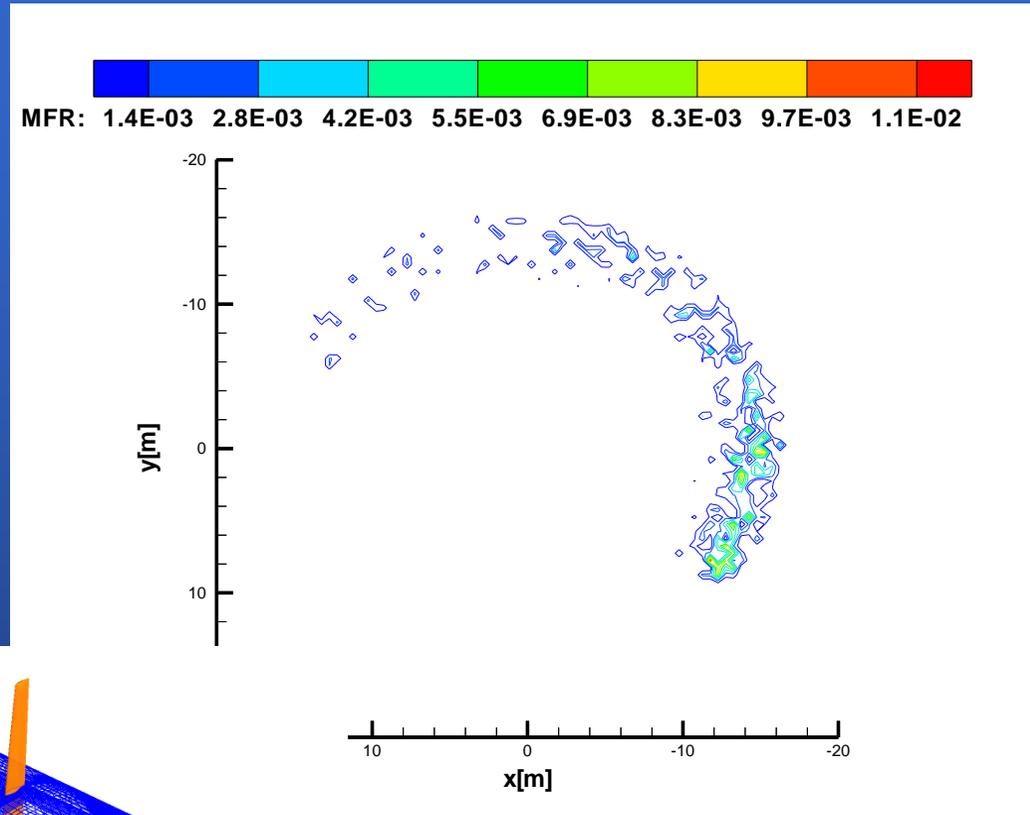
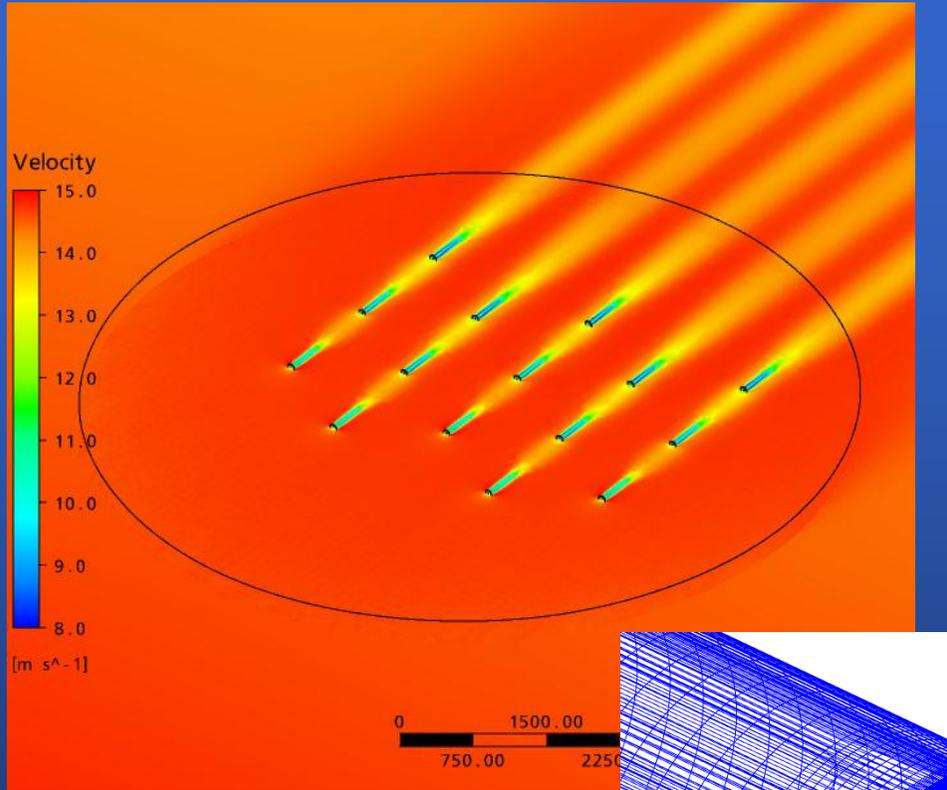
World class engineering expertise  
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# Areas of expertise and experience



# Areas of expertise and experience



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# Introduction

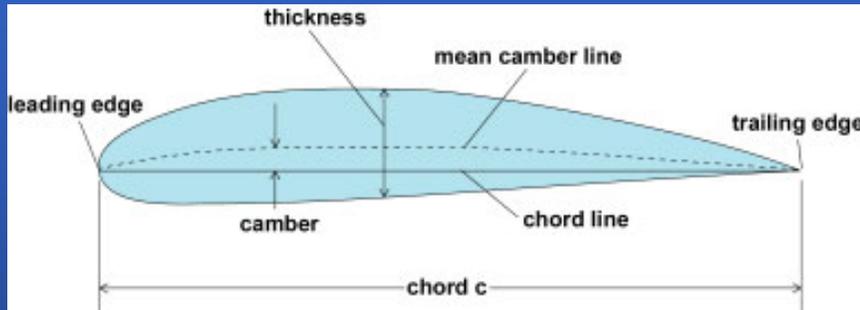
Global energy demands and dwindling fossil fuels are making zero-head wind and water turbines increasingly important as renewable energy devices

- **Device efficiency** is a key parameter in the economic viability, recovery of initial capital investment and long term profitability of such devices.
- To determine a performance curve of an operational turbine, a range of aspects need to be taken into account. These include **complex fluid dynamics**, **local topology** and other **environmental conditions**, **structural loadings** and **vibrations**, **transmission loads** and **generator dynamics**.
- Furthermore, in most cases, turbines are installed in groups or farms. Flow interaction in a **multiple turbine installation** reduces the power output and thus has an important influence on economics of whole installations.



# Introduction

Design of the modern wind turbine rely on the same **aerodynamic principles** as an aircraft



Wind turbines fall into two main categories, those that depend upon **aerodynamic drag** to drive them (i.e. old style windmills) and those that depend upon **aerodynamic lift**.



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# Introduction

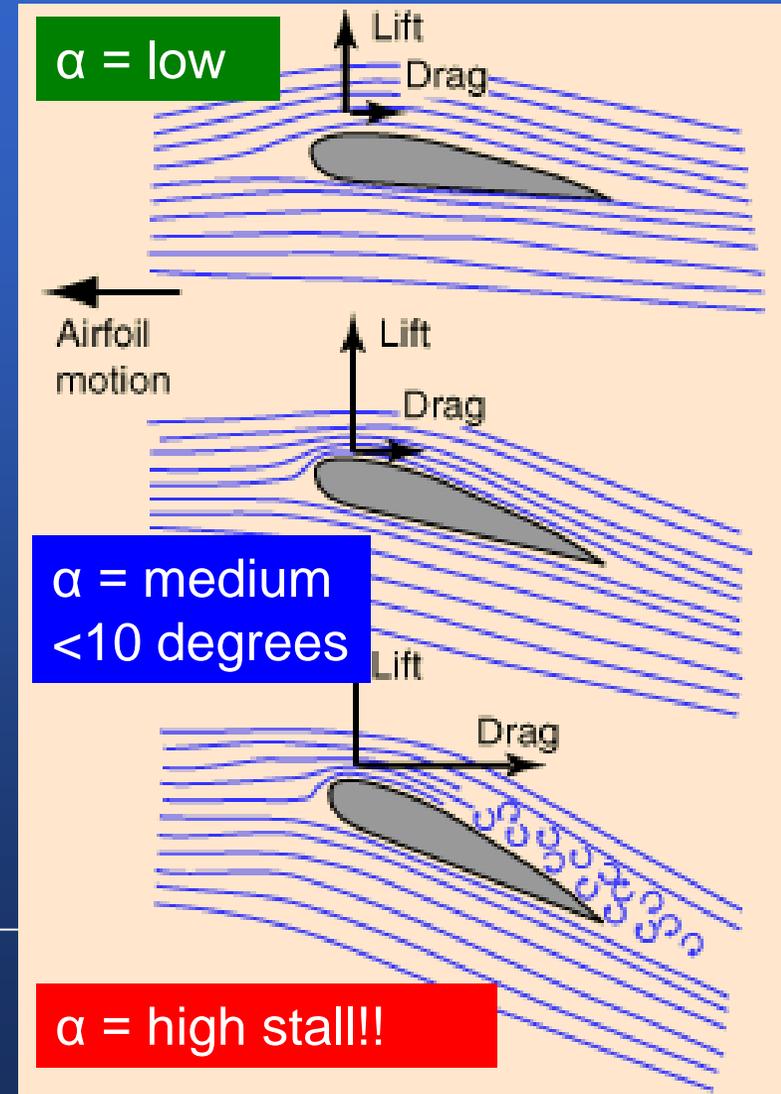
The lift and the drag forces strongly depend on the incidence angle

- **Lift Force** is perpendicular to the direction of motion
- **Drag Force** is parallel to the direction of motion

Modern wind turbine design - the **lift force is maximized**, whereas the developed drag force stays small (1 to 2% of the lift force in pre-stall conditions)



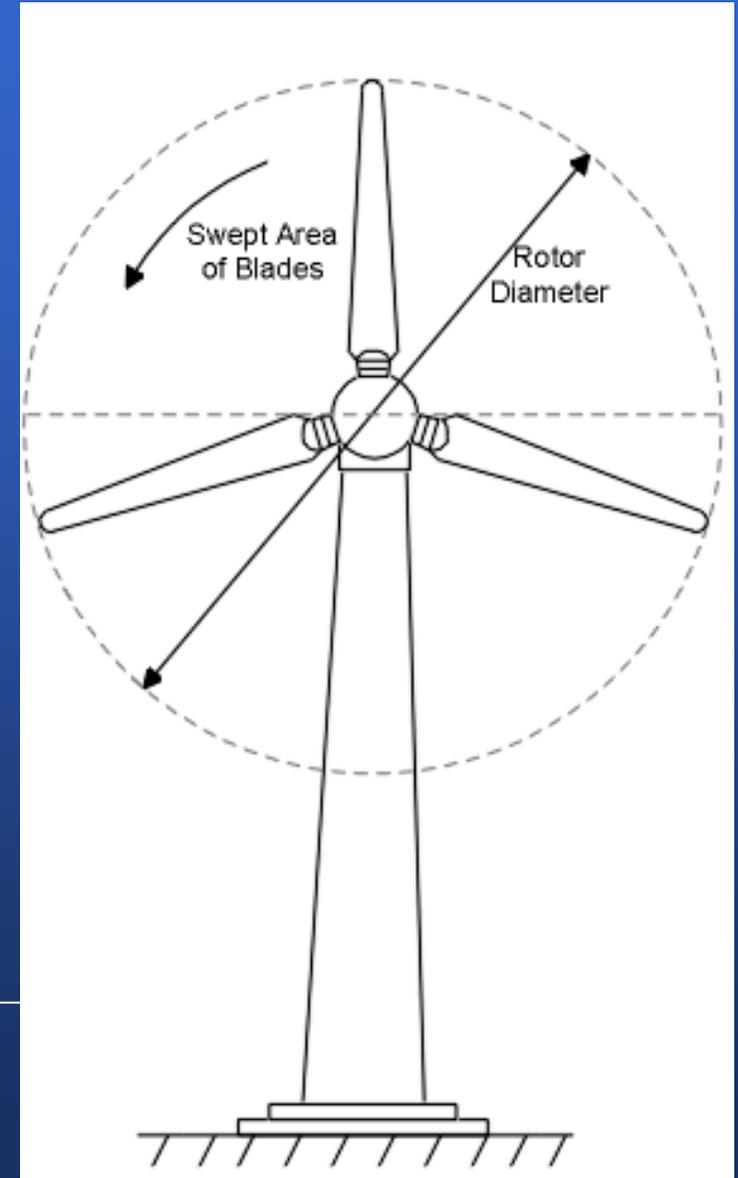
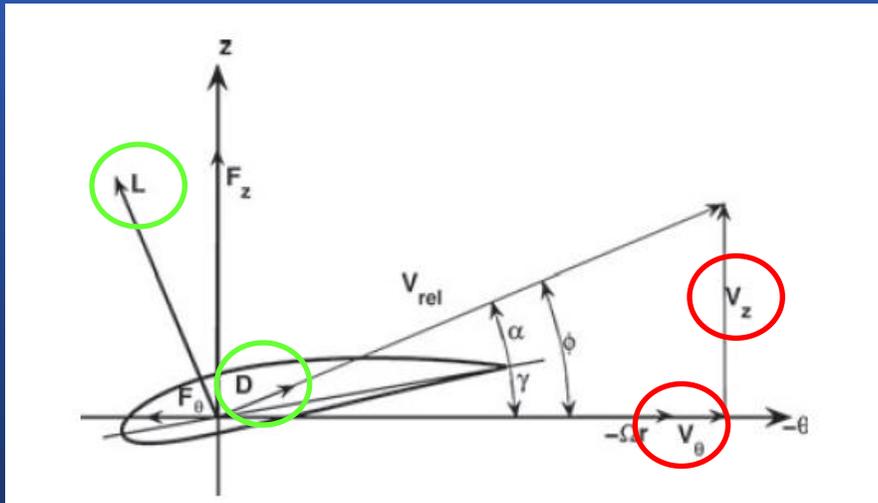
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# Introduction

The turbine rotates with angular velocity  $\Omega$  and tangential velocity  $\Omega r$

Wind with local velocity components  $v_z$  and  $v_\theta$  creates lift force  $L$  and drag force  $D$ , which at a correct incidence angle  $\alpha$  turn around the turbine rotor



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courtesy of ESN

# Theoretical turbine prediction models

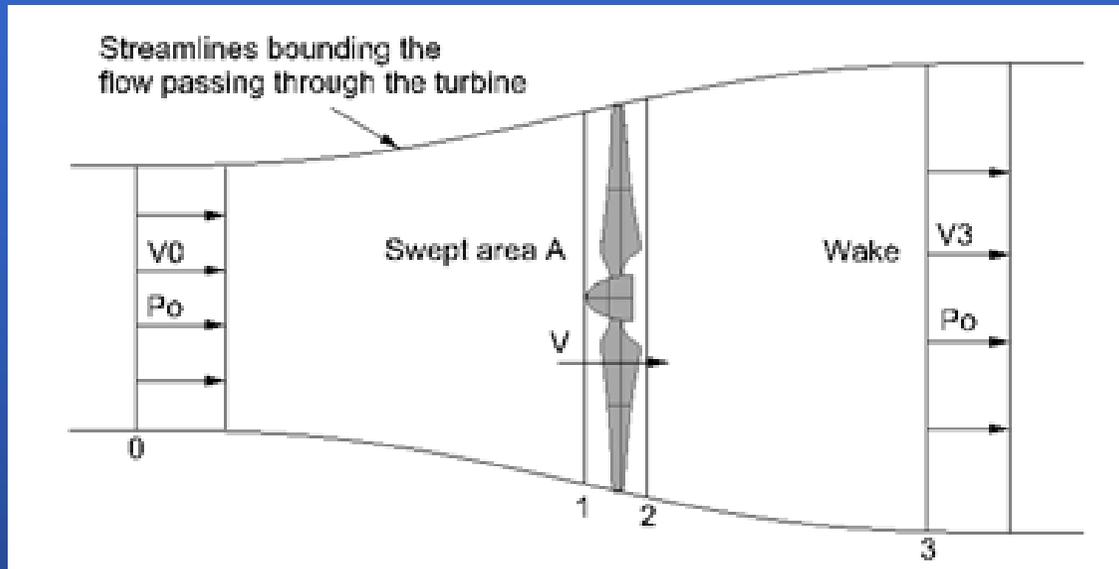
The simplest model (Betz, 1926) that can be used to predict wind turbine power output (and, subsequently, turbine efficiency) represents the rotor as an **actuator disc**, which creates a pressure discontinuity i.e. acts as a momentum sink

The **Actuator Disc** approach uses the following assumptions:

- The flow is ideal and rectilinear across the turbine i.e. steady, homogenous, inviscid, irrotational, and incompressible
- Both the flow and thrust are uniform across the disk
- The static pressure at the upwind and downwind boundaries is equal to the ambient static pressure



# Theoretical turbine prediction models



$$P = F_a v$$

$$P = \dot{m}(v_0 - v_3)v = (p_2 - p_1)A$$

Energy loss by the wind is

$$P = \dot{m}(v_0^2 - v_3^2)/2$$

and therefore  $v = (v_0 + v_3)/2$

- The wind exerts an axial thrust on the turbine in the flow direction
- An equal and opposite force is exerted by the turbine on the wind through its mounting with the ground



# Theoretical turbine prediction models

More often the following form is derived<sup>2</sup>

$$P = 2\dot{m}(v_0 - v)v = 2 Av^2(v_0^2 - v^2)$$

The fractional decrease in wind velocity between the free stream and rotor plane can be expressed in terms of an axial induction factor  $a$ :

$$a = (v_0 - v)/v_0 \quad \text{[?]}$$

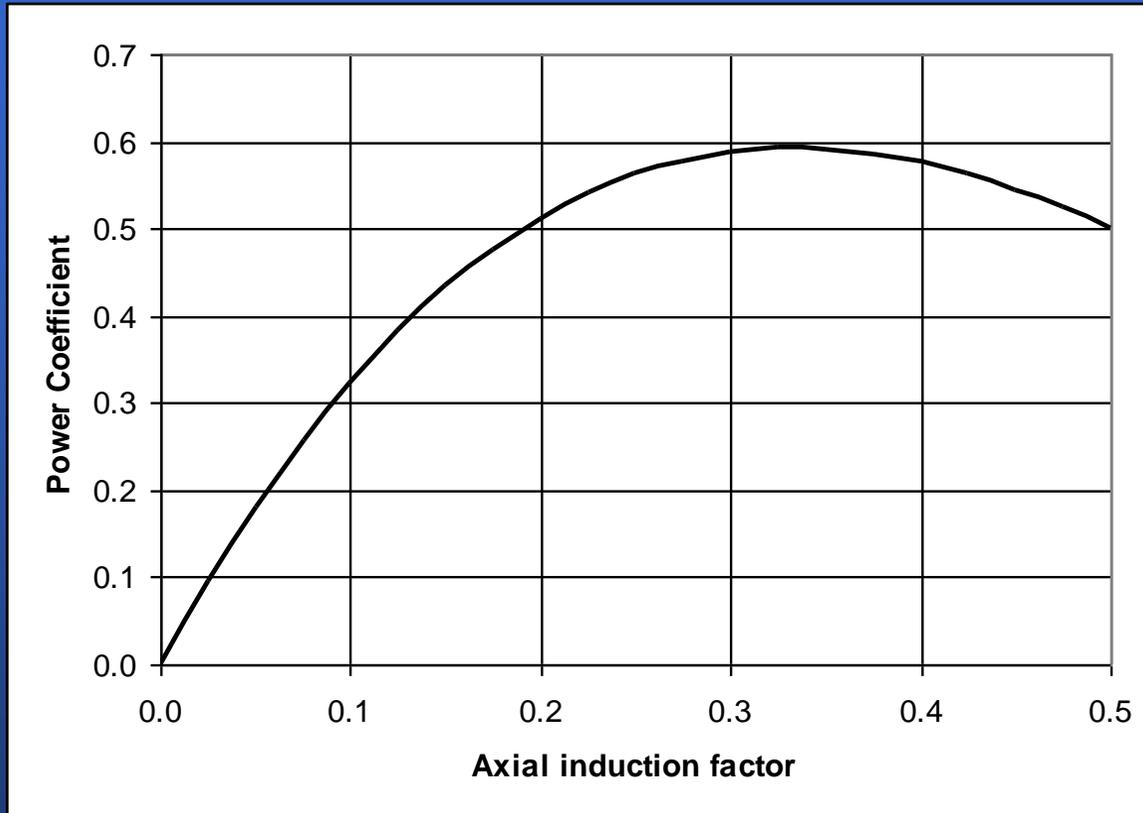
Then the expression for turbine power output is  $P = 2 Av^3 a(1 - a)^2$

and the power coefficient (i.e. the extracted power over the total available power) is simply

$$C_P = \frac{P}{\frac{1}{2} \dot{m} v_0^2} = 4a(1 - a)^2$$



# Theoretical turbine prediction models



The maximum value of  $a = 1/2$ ;  
requires  $v_3 \rightarrow 0$

$C_p$  reaches maximum of 0.593  
when  $a = 1/3$

The corresponding downstream  
wake velocity,  $v_3 = v_0/3$  and  
the wake area  $A_3$  is double the  
turbine swept area  $A$

This is known as the Betz limit  
for an ideal frictionless turbine



# Theoretical turbine prediction models

All wind power cannot be captured by rotor or air would be completely still behind rotor

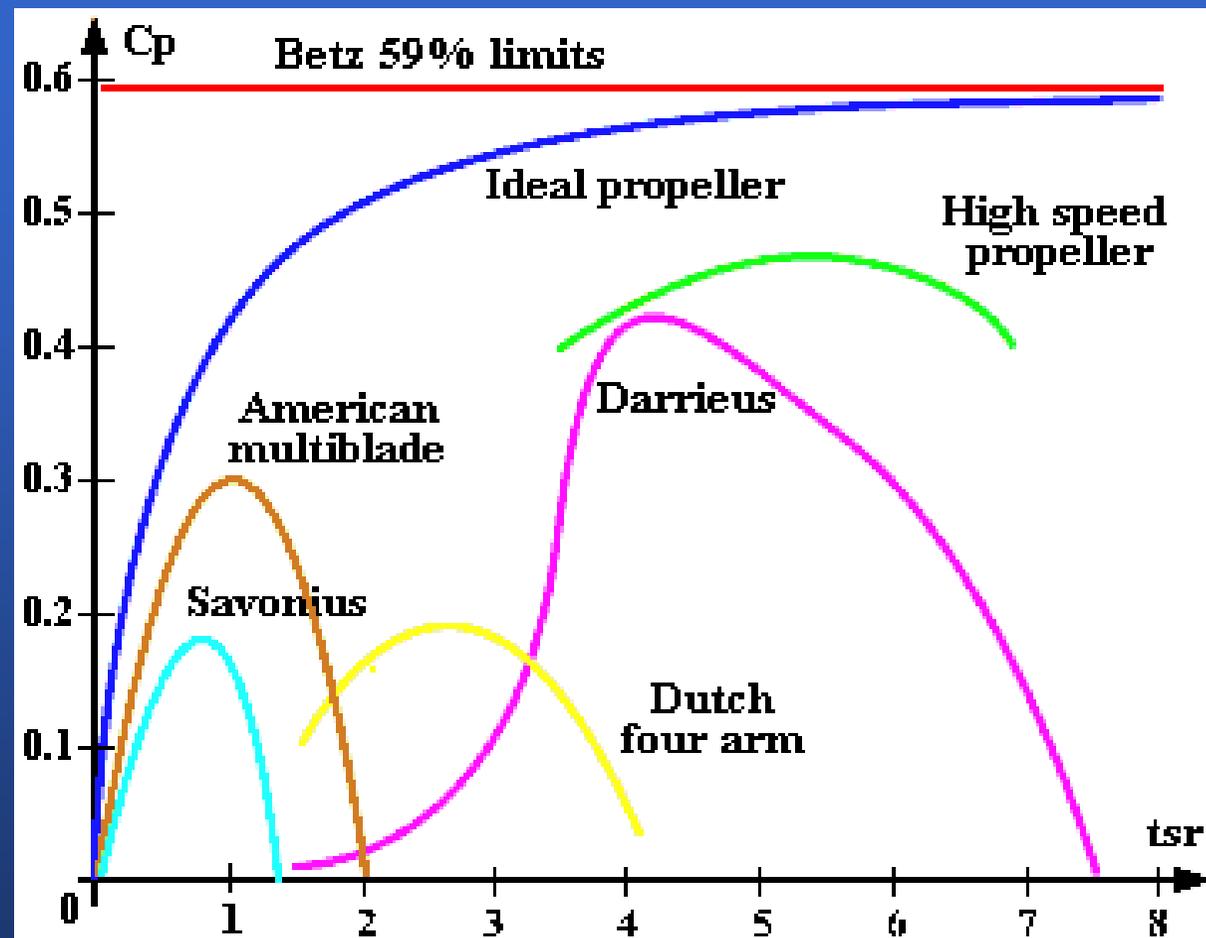
Theoretical limit of rotor efficiency is 59%

In reality a real wind turbine does not achieve this efficiency level due to:

- Rotation in the wake caused by the reaction with the spinning rotor
- A non-uniform pressure distribution in the turbine plane
- Aerodynamic drag due to viscous effects
- Energy loss due to vortices at the blade tips



# Theoretical turbine prediction models



# Theoretical turbine prediction models

The impact of the rotating wake can be estimated by extending the Betz analysis to a 2-D model in the radial direction (Glauert, 1935)

The control volume is divided into many non-interacting elementary radial blade sections that could be analysed independently (Blade Element Method)

- The flow far upstream is purely axial; however there is a discontinuous jump in angular velocity across the rotor plane because torque is exerted on the rotor
- The turbine wake rotates in the opposite direction to the rotor with an angular velocity  $\omega_2$
- Circumferentially averaged torque on a radial element is then

$$d\tau = \omega_2 r^2 d\dot{m} = \omega_2 r^2 (\rho v_2 2\pi r dr)$$



# Theoretical turbine prediction models

- Blade element model usually utilizes an angular induction factor and a local speed ratio:

$$a' = \frac{\omega_2}{2\Omega} \quad \lambda_r = \frac{\Omega r}{v_0}$$

- The turbine torque and power are calculated by integrating radial contributions

$$\tau = 4\pi\rho\Omega v_0 \int_0^R (1-a)a'r^3 dr \quad P = \tau\Omega$$

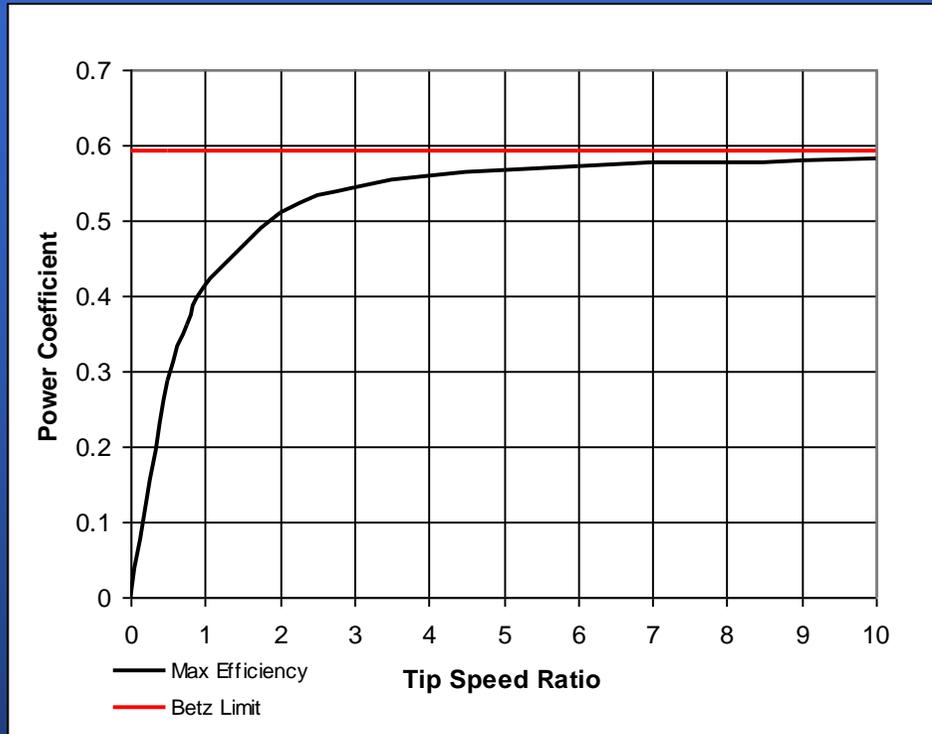
where  $a$  and  $a'$  are radial functions, which need to be empirically determined

- Also, it can be shown for an ideal turbine the the axial and angular induction factors are related

$$a' = \frac{1}{2} \left( \sqrt{1 + \frac{4a}{\lambda_r^2} (1-a)} - 1 \right)$$



# Theoretical turbine prediction models



Power coefficient  $C_p$  when wake rotation is taken into consideration

Turbines with high tip-speed ratio (TSR) and low angular induction factor ( $a'$ ) are able to deliver more power



# Theoretical turbine prediction models

- Glauert's wake rotation model is still subject to the assumptions of a uniform distribution in tangential direction and zero radial velocity in the turbine plane
- Due to these assumptions, the model overestimates forces and torque applied to the turbine
- Therefore, Glauert's model asymptotes towards the Betz limit as the tip-speed ratio tends to infinity, i.e. as the rotation in the wake tends to zero

More recently, in 2001, Gorban, Gorlov and Silantyev introduced an alternative model, that considers non-uniform pressure distribution and curvilinear flow across the turbine plane



# Theoretical turbine prediction models

One of the variations of the approach is the **Actuator Line Model**:

- The blade relative velocity  $v_{rel}$  and incidence angle  $\alpha$  are calculated from the local wind velocity components

$$v_{rel} = \sqrt{v_z^2 + (\Omega r - v_\theta)^2} \quad \alpha = \varphi - \gamma$$

where relative flow angle is  $\varphi = \tan^{-1} \left( \frac{v_z}{\Omega r - v_\theta} \right)$  and  $\gamma$  is local blade pitch angle

- Based on local values of Reynolds number ( $Re$ ) and the incidence angle ( $\alpha$ ), lift ( $C_L$ ) and drag ( $C_D$ ) coefficient are selected from a predefined database
- Lift and drag force per spanwise length of the blade are then expressed as

$$F_L = \frac{1}{2} C_L \rho |v_{rel}| v_{rel} l \quad F_D = \frac{1}{2} C_D \rho |v_{rel}| v_{rel} l$$



# Theoretical turbine prediction models

- Axial and tangential force acting on a blade segment are defined as

$$F_z = F_L \cos(\varphi) + F_D \sin(\varphi) \quad F_\theta = F_L \sin(\varphi) - F_D \cos(\varphi)$$

- The torque produce by each blade is then obtain by integration in the radial direction

$$\tau = \int_0^R r F_\theta dr$$



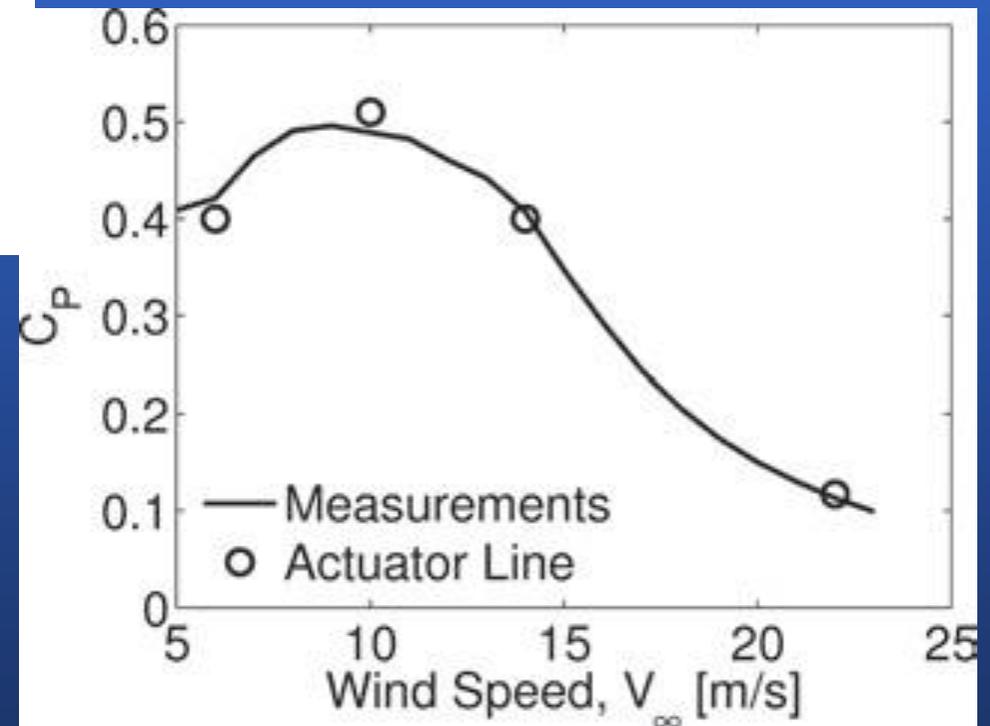
# Theoretical turbine prediction models



Wake visualisation using vorticity isosurface

Power coefficient comparison for the Tjareborg wind turbine

N. Troldborg et al.  
(Wind Energ., 2009)



# Computational Fluid Dynamics (CFD) approach

- Solves discretized and averaged Navier-Stokes equations for the flow around turbine blades
- Due to turbulence motion, the transport equations are averaged over time (RANS turbulence models) or filtered using a spatial filter (LES turbulence models)

$$\partial_t \bar{\rho} + \partial_j (\bar{\rho} \tilde{v}_j) = 0$$

$$\partial_t (\bar{\rho} \tilde{v}_j) + \partial_i (\bar{\rho} \tilde{v}_i \tilde{v}_j) = -\partial_j \bar{p} + 2\partial_i (\mu \overline{S_{ij}}) + \bar{\rho} g_j - \partial_i (\bar{\rho} \Pi_{ij})$$

$$\bar{\rho} \Pi_{ji} = \overline{\rho v_j v_i} - \bar{\rho} \tilde{v}_j \tilde{v}_i \Rightarrow \overline{\rho v_j^* v_i^*}$$

$$\bar{\rho} \Pi_{ij} - \frac{1}{3} \bar{\rho} \Pi_{ll} \delta_{ji} = -2\mu \overline{S_{ij}} + \frac{2}{3} \mu_t (\partial_i \tilde{v}_l) \delta_{ji}$$

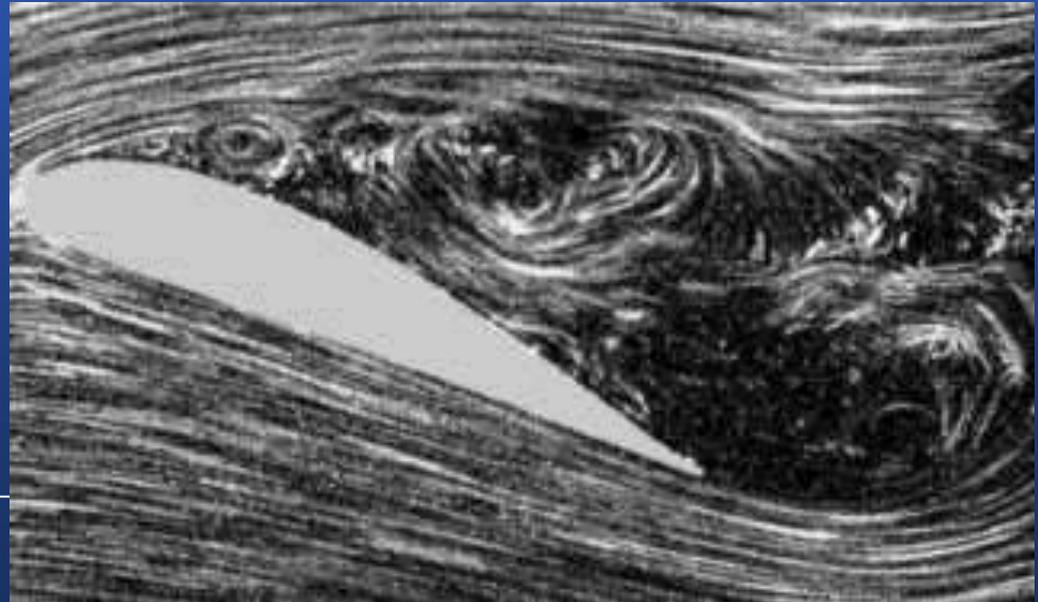
Turbulent stresses, in most cases modelled using the eddy viscosity concept



# Computational Fluid Dynamics (CFD) approach

CFD can provide three-dimensional and time variation of all flow parameters, nevertheless, accuracy of the numerical prediction crucially depends on

- Resolution of the numerical grid i.e. number of computational points and their distribution
- Suitability of turbulence model to capture complex flow behaviour i.e. boundary layer transition, separation, transient effects, trailing and tip vortices



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Jaganath (GNU)

# Computational Fluid Dynamics (CFD) approach

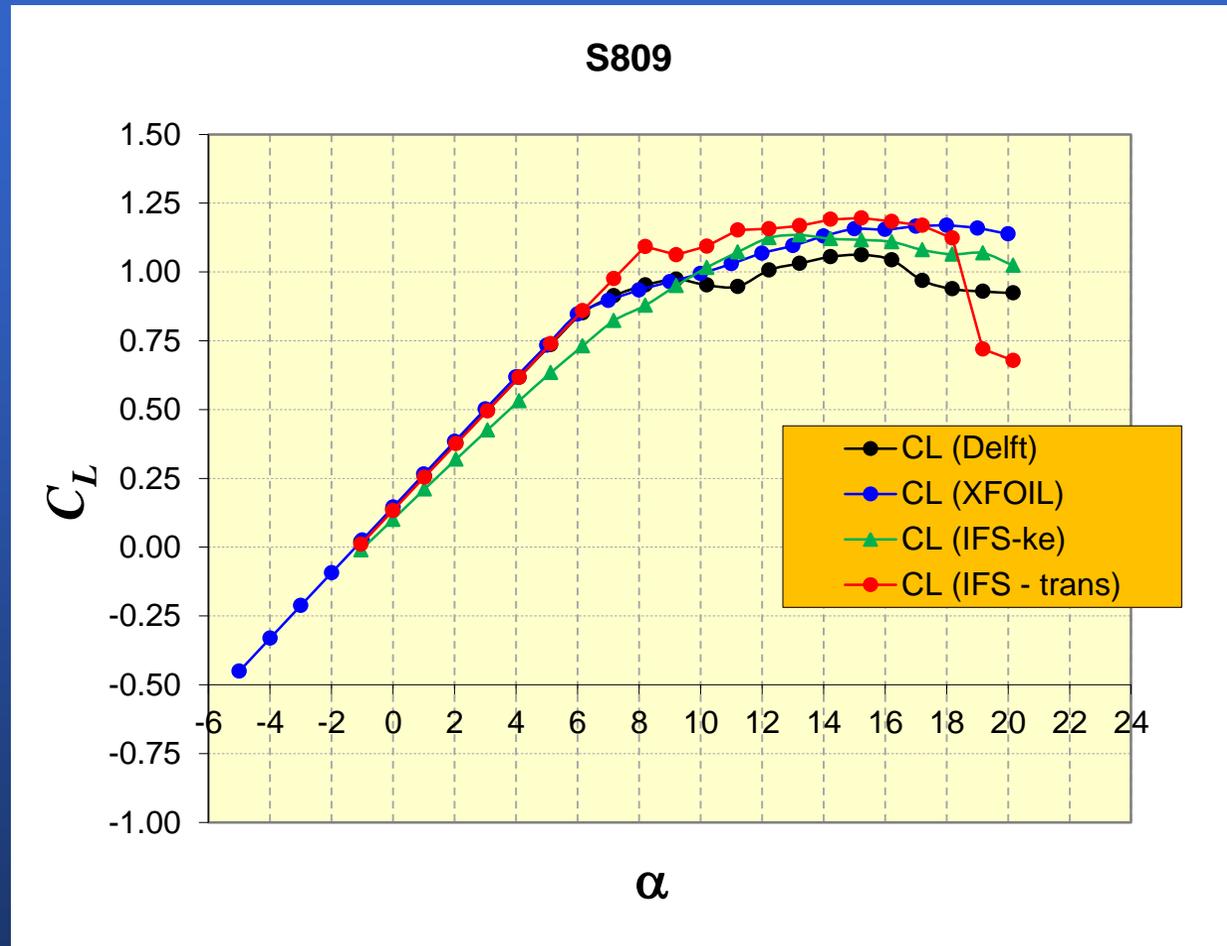
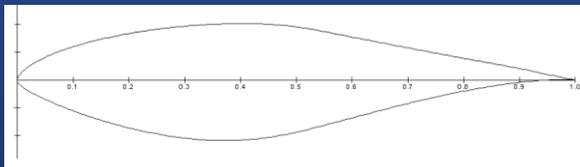
The required computational resources for CFD simulations of a turbine during its operation are extremely large, therefore, different simplifications and empirical models are introduced or coupled with the CFD model:

- Two dimensional CFD simulations of a wing section may be utilized to calculate drag and lift force (or drag and lift coefficient) for different incidence angles
- Such database  $C_L(Re, \alpha)$  and  $C_D(Re, \alpha)$  can then be used to integrate forces along the blade and to calculate torque and the turbine power output



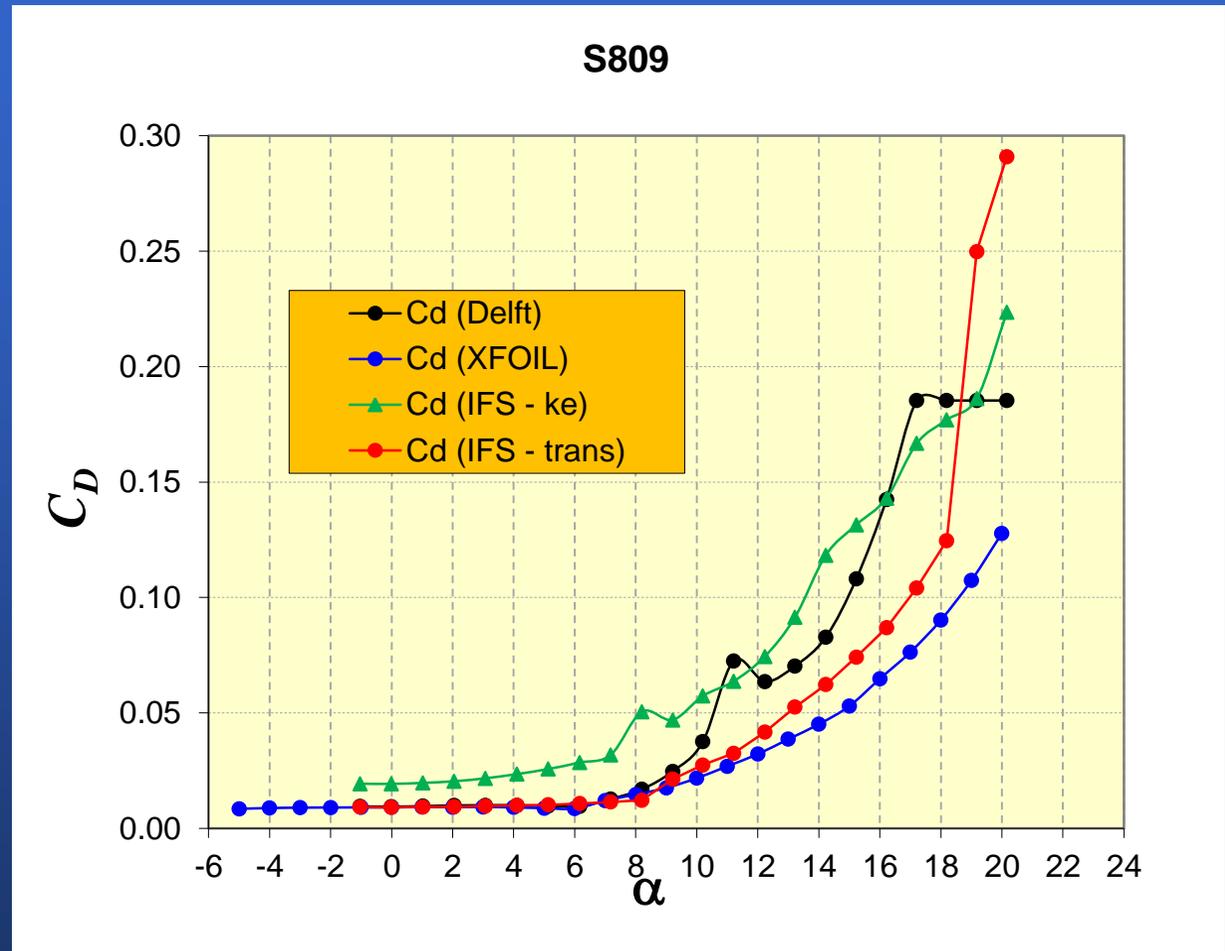
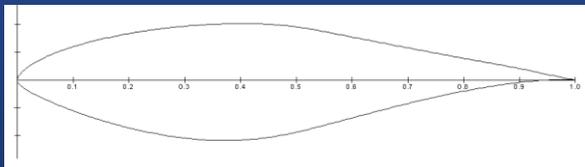
# Computational Fluid Dynamics (CFD) approach

Lift coefficient comparison  
for airfoil S809



# Computational Fluid Dynamics (CFD) approach

Drag coefficient comparison  
for airfoil S809



# Computational Fluid Dynamics (CFD) approach

The 2D CFD simulations of turbine segments cannot take into account

- Radial flow effects
- Influence of the nacelle and the support tower on the flow behaviour
- Three-dimensional effects of the wake formation
- Surrounding topology
- Environmental effects – ground surface boundary layer formation, buoyancy, flow stability etc

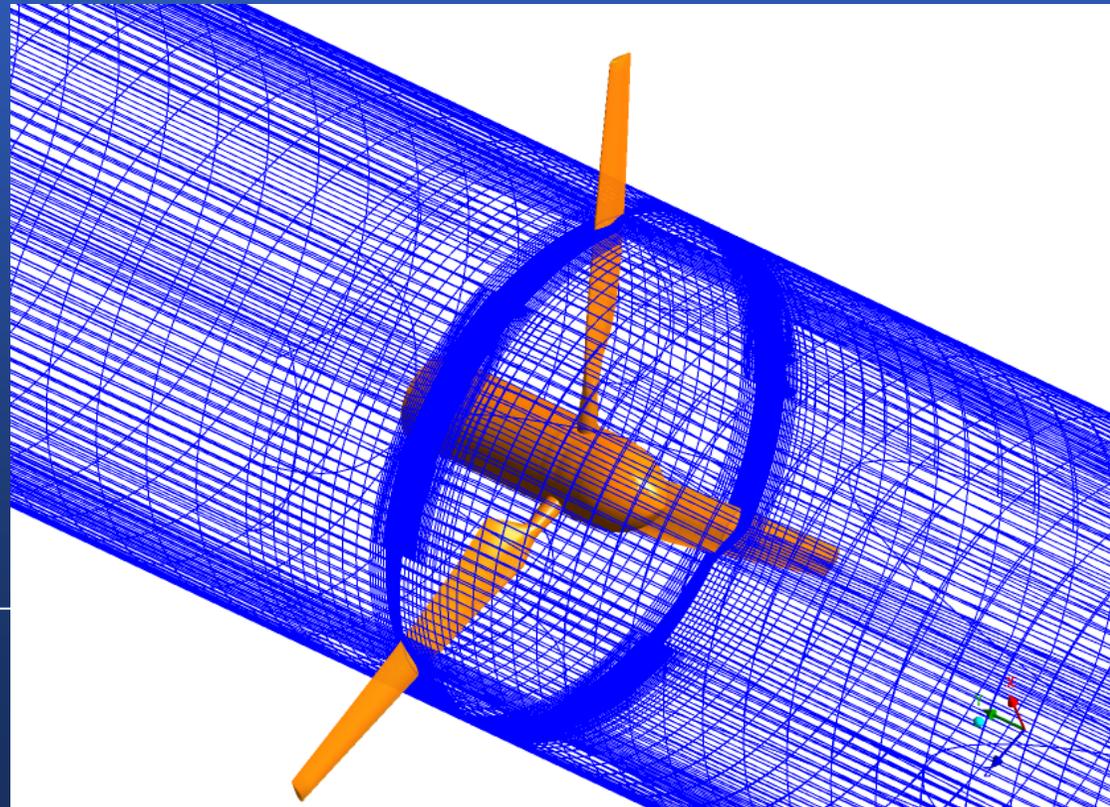


# Computational Fluid Dynamics (CFD) approach

Three dimensional CFD simulations do overcome deficiencies of 2D model, although they are rarely performed for engineering purposes:

- require numerical grids of at least few million of nodes
- due to transient flow effects, they need to be run transiently with significantly small timestep

Section of the numerical mesh for UAE turbine

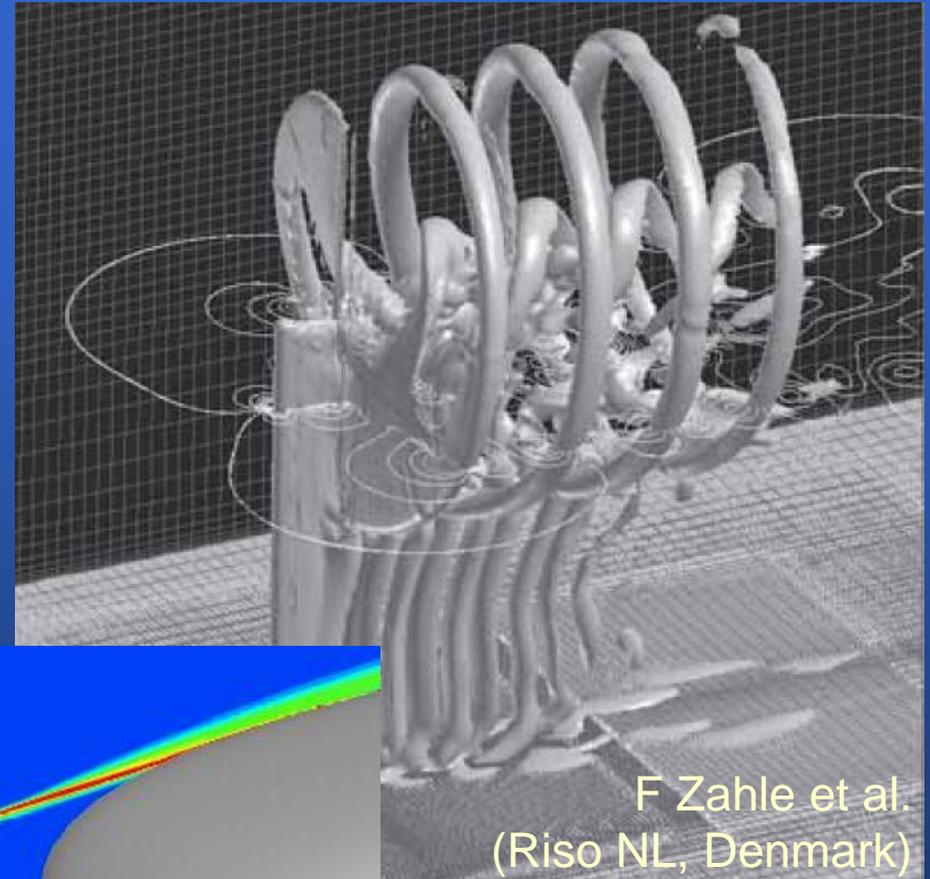


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# Computational Fluid Dynamics (CFD) approach

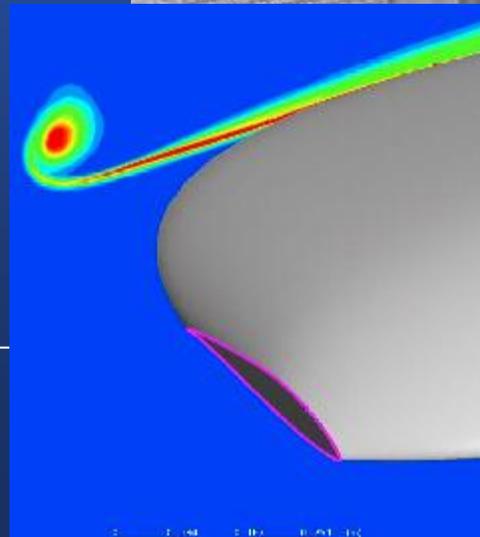
Three dimensional CFD simulations allow investigation of

- Blade design details (radial profile and pitch changes)
- Influence of nacelle and support tower
- Difficult flow physics conditions (tip vortex, stall conditions etc)



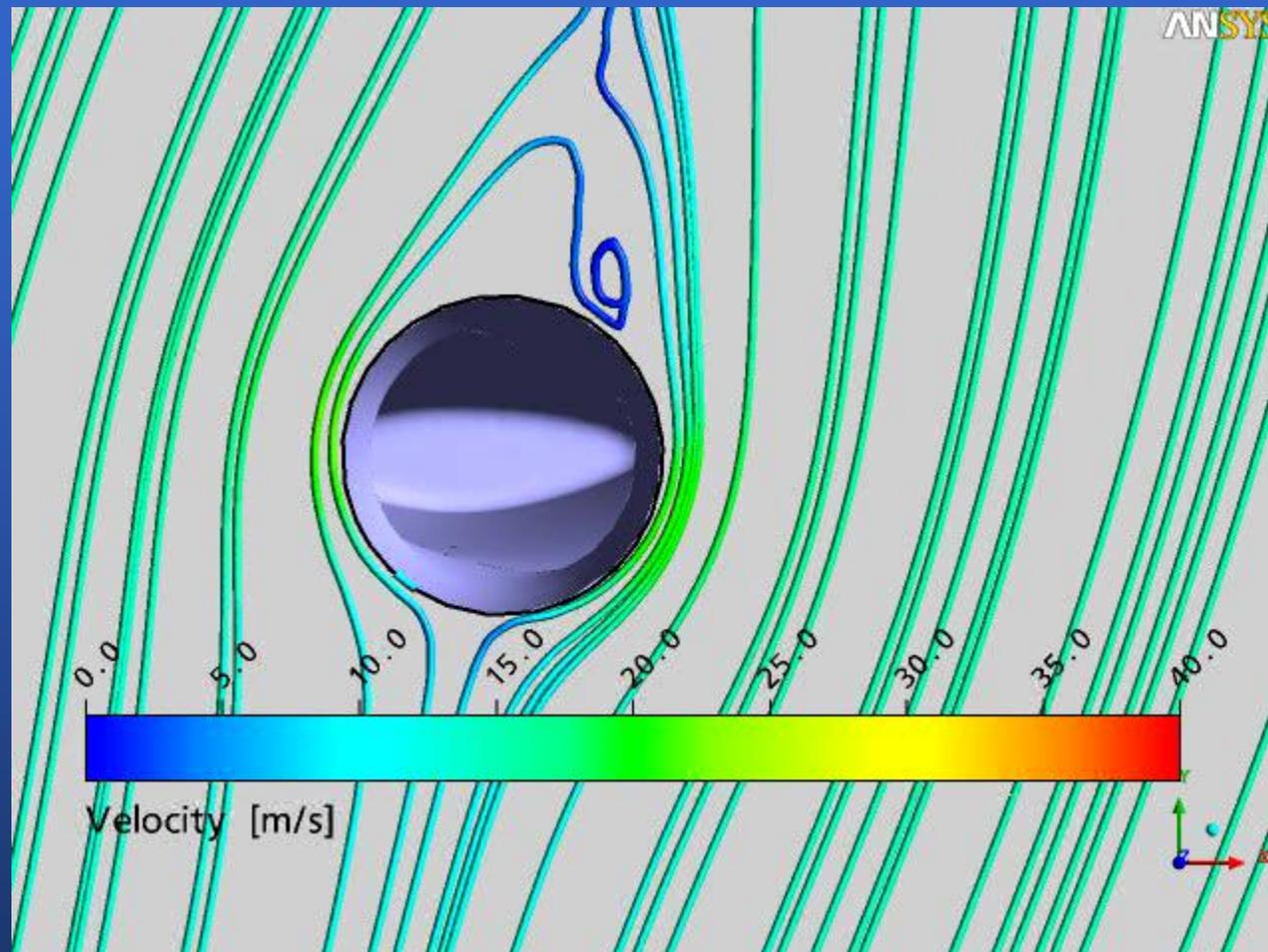
F Zahle et al.  
(Riso NL, Denmark)

S. Herr et al.  
(GE Wind Energy)



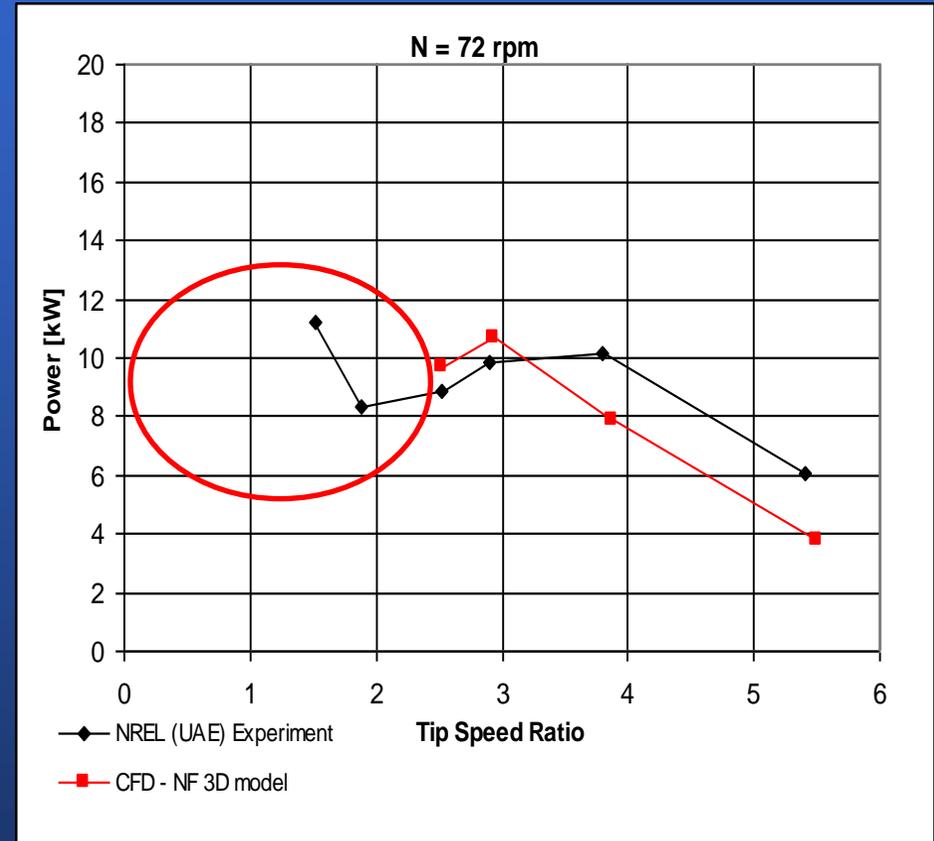
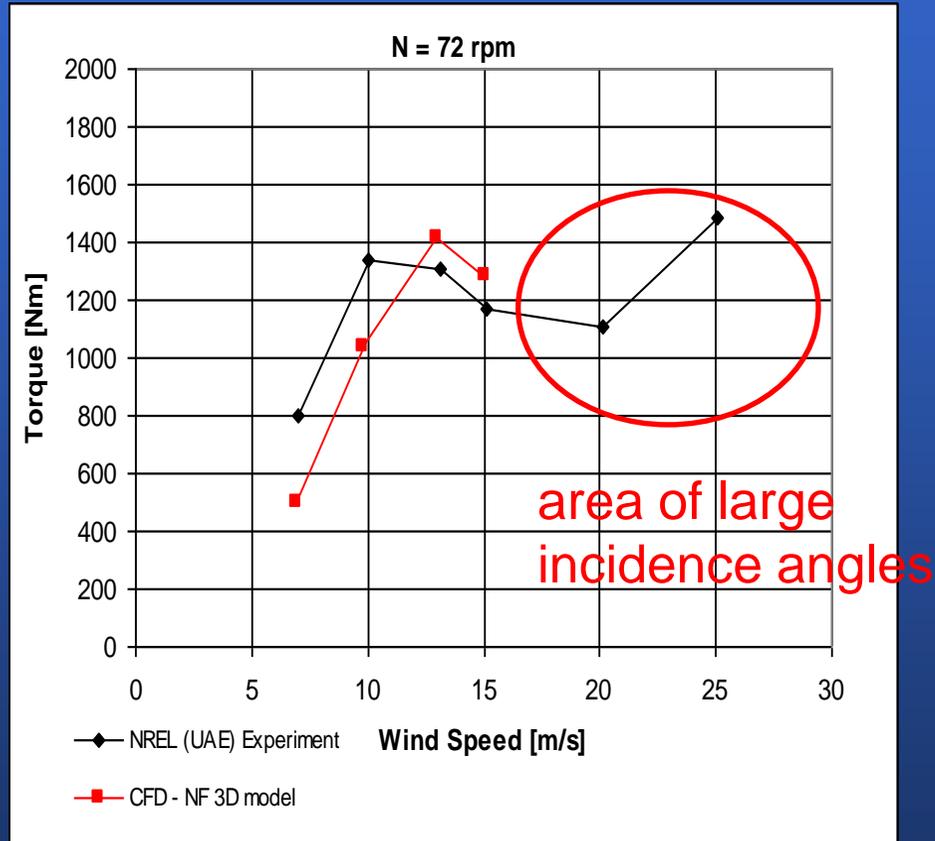
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# Computational Fluid Dynamics (CFD) approach

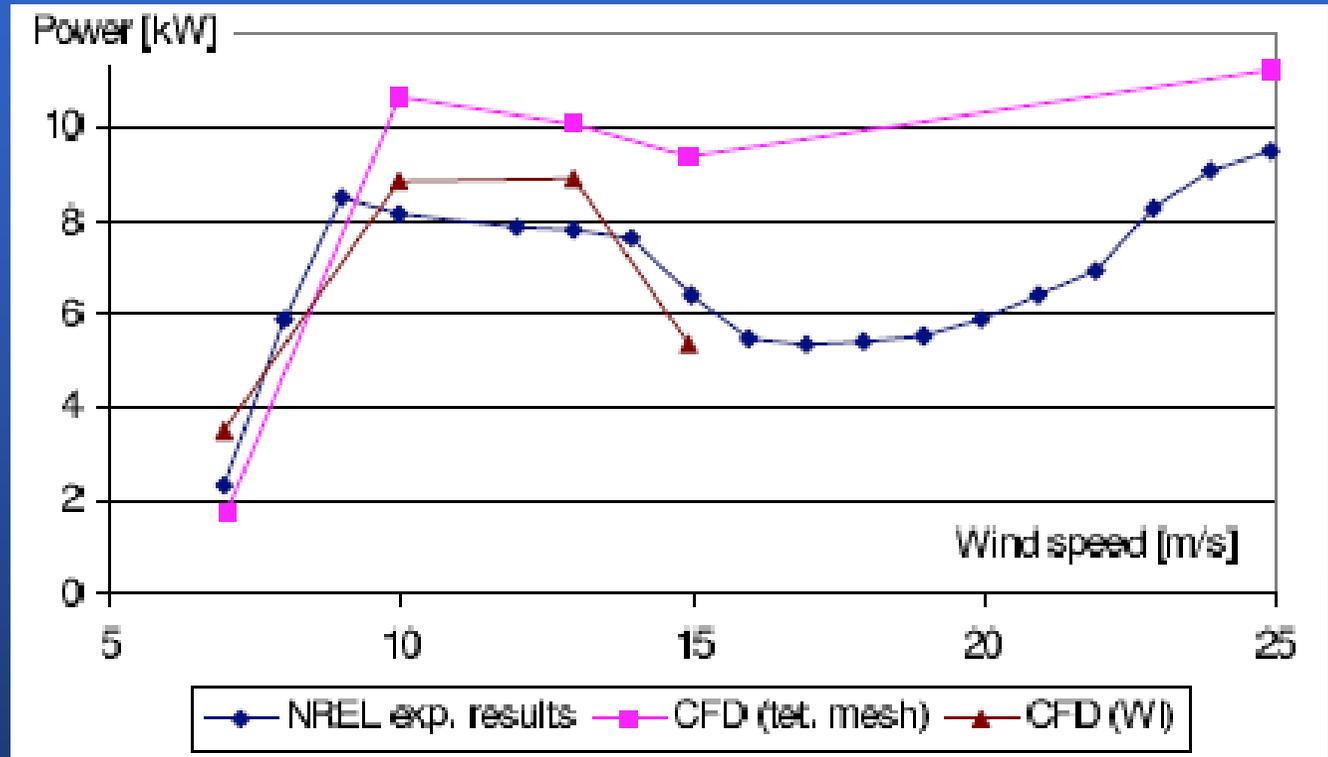


# Computational Fluid Dynamics (CFD) approach

How accurate are flow predictions using 3D CFD modelling ?



# Computational Fluid Dynamics (CFD) approach



S. Herr et al.  
(GE Wind Energy)



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# Wind farm modelling and optimization

Accurate prediction of wind farm power output is necessary for planning installation capacity and to **maximise return on the investment**

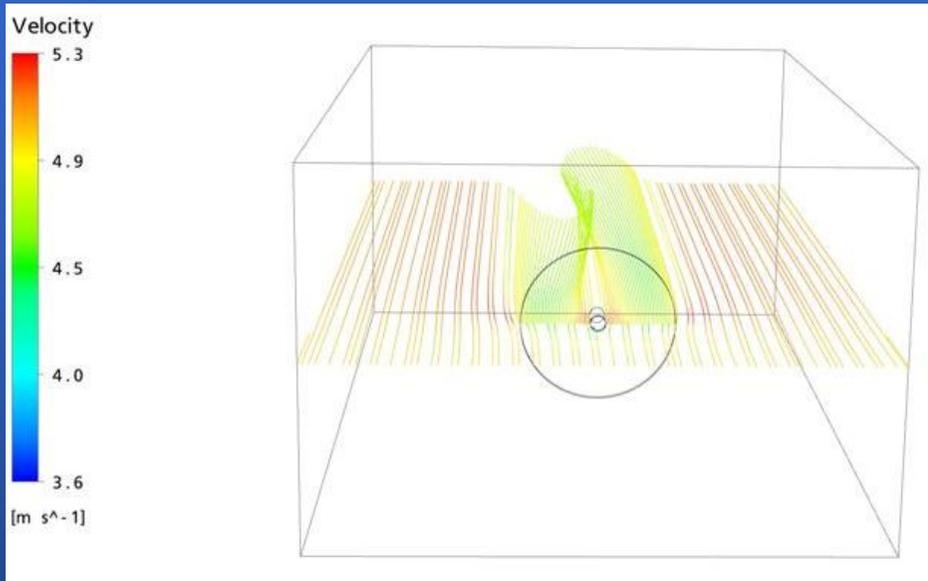
Beside flow physics associated with individual turbines, modelling need to include

- **wake interaction**
- **topology**
- **environmental conditions (i.e. ground boundary layer, buoyancy effects)**

Combination of **CFD approach** and **actuator disc** (or **blade element**) model is necessary to reduce computational requirements



# Wind farm modelling and optimization

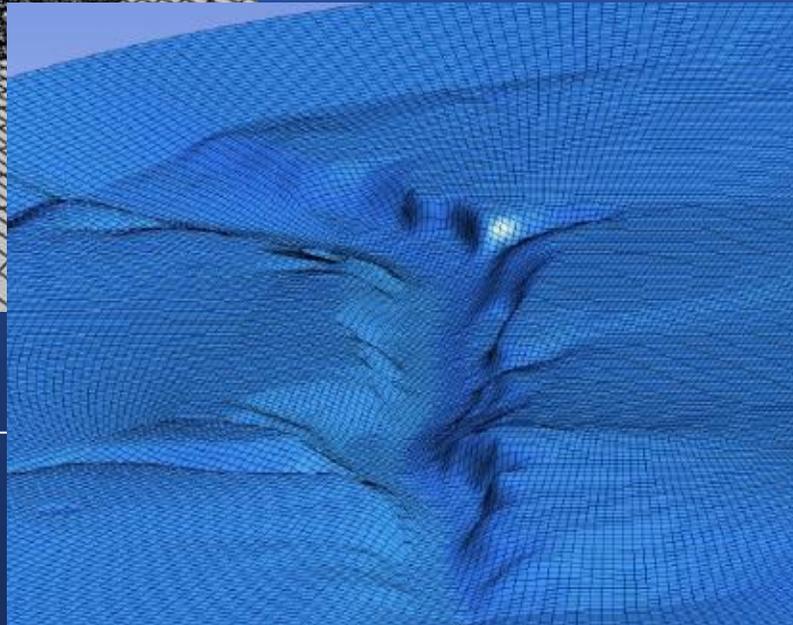
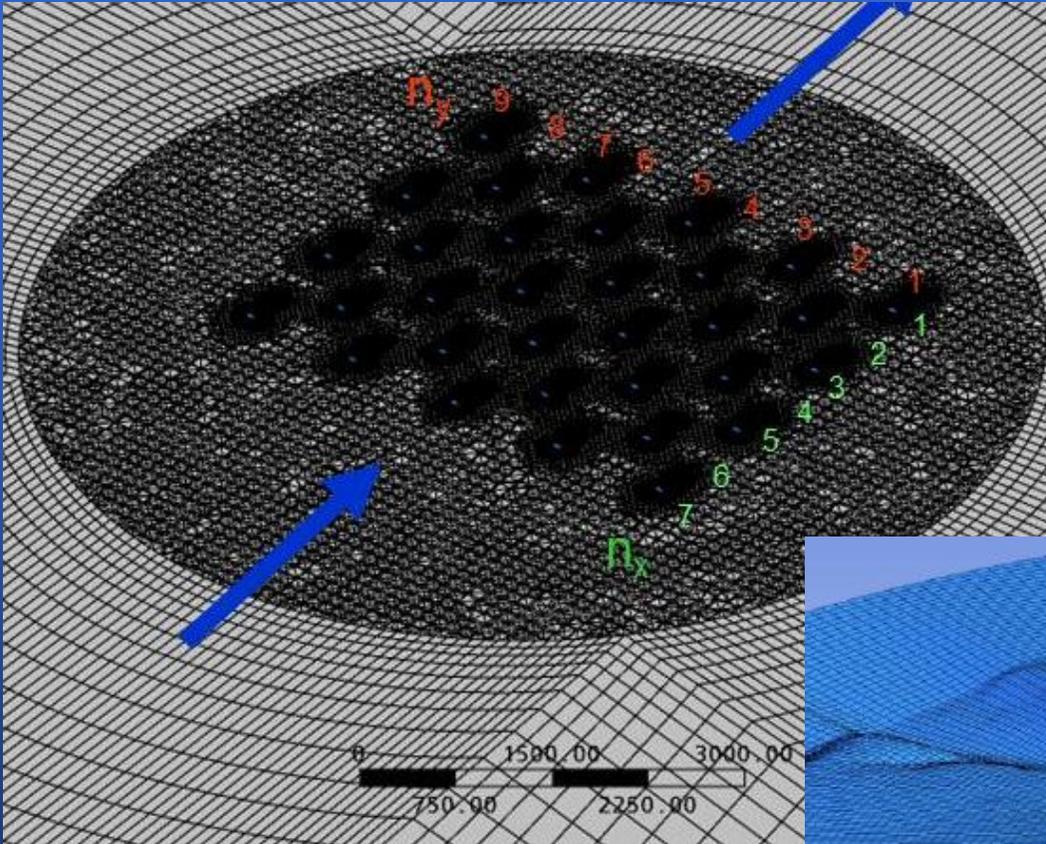


- Based on local velocity distribution, **actuator disc** (or blade element) model calculates forces and power output
- Effects of a rotating turbine on the flow are modelled with **momentum sources/sinks**
- Correct time-averaged representation of axial and tangential wake velocities



# Wind farm modelling and optimization

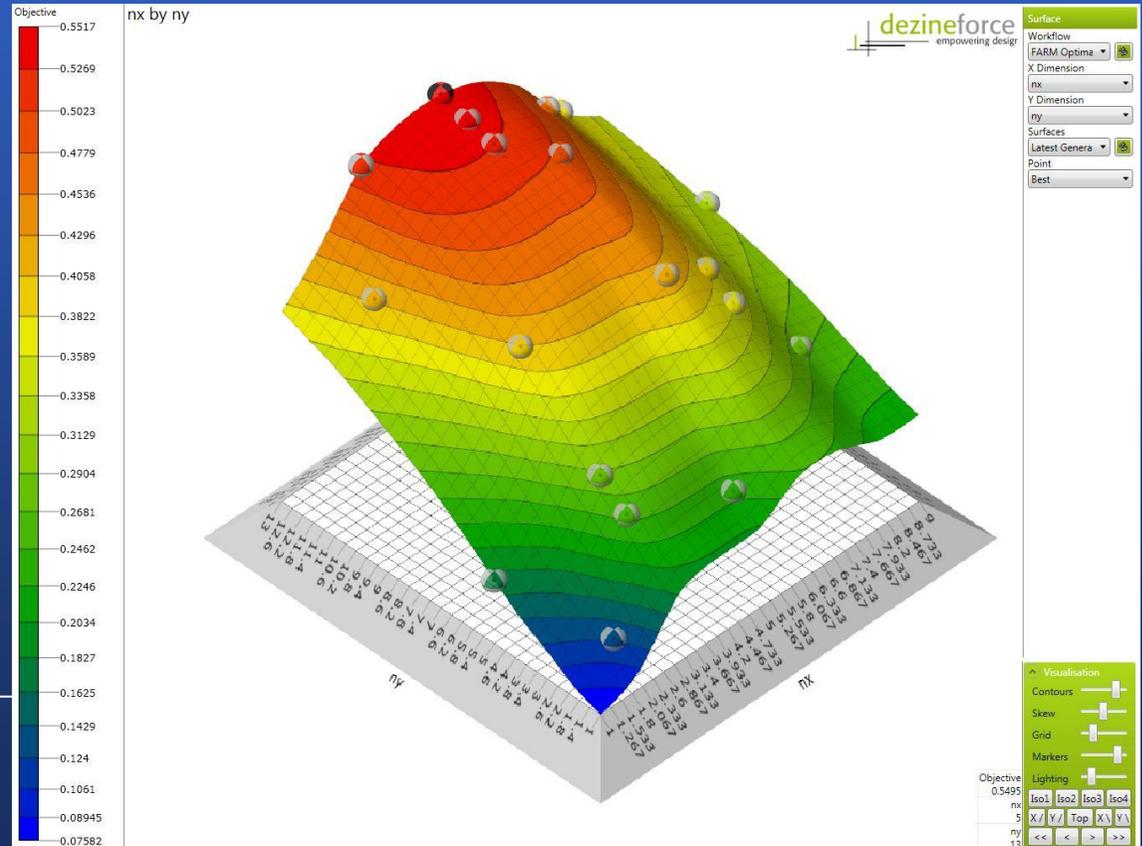
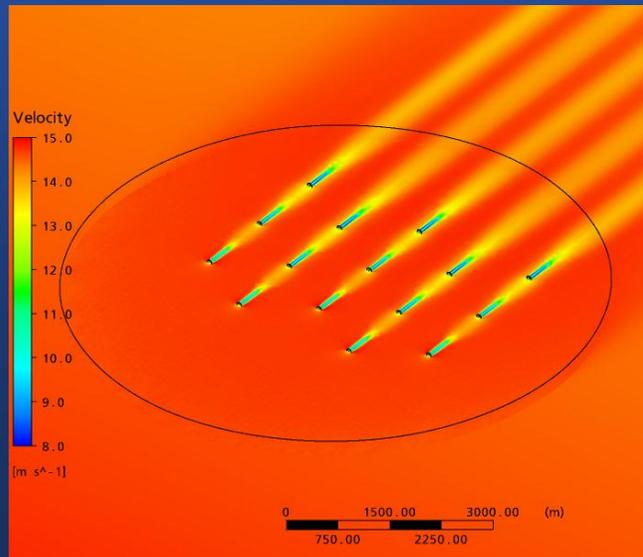
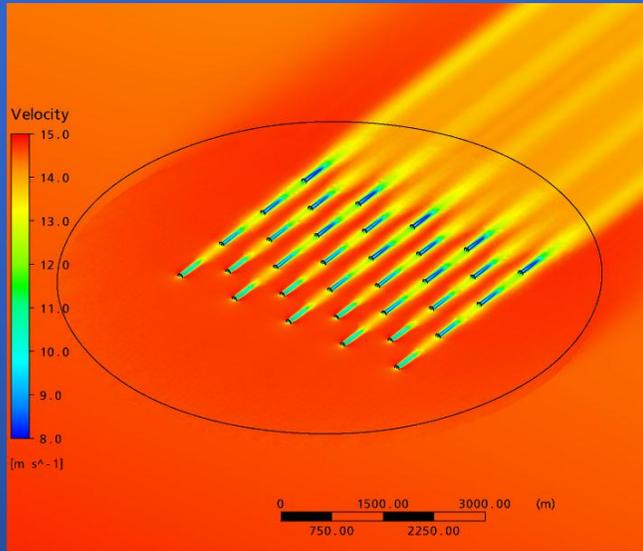
- The turbine (actuator disc) model is then inserted in a **CFD model** of a wind farm
- **Different turbine arrangements** can be analysed to minimize wake effect
- **Real topology** environment can be assessed



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# Wind farm modelling and optimization

- Such combine model can be coupled with an optimizer to maximize power output for a given investment



# Summary

- The design of modern wind turbines rely on the same **aerodynamic principles** as an aircraft - predicting **lift** and **drag** force in a challenging environmental conditions
- **Actuator Disc (Betz)** approach gives an approximation of the turbine maximum power output
- **Blade Element (Glauert)** model introduces radial variation of the flow condition and wake rotation, where power output is related to the turbine tip-speed ratio
- Although useful as a first approximation, both models overestimate the power output
- **Actuator Line model** relies on **databases** of the lift and drag coefficient as functions of local flow conditions
- The accuracy power prediction depends on the accuracy of utilized databases



# Summary

- Using **Computational Fluid Dynamics (CFD)** modelling, lift and drag forces (or lift and drag coefficient) are calculated from local flow and geometrical conditions
- **Flow physics** is captured much more accurately, which considerably improves accuracy of the turbine power output prediction
- Despite advance turbulence modelling, **prediction of stall conditions** is still (at least) **challenging**
- Especially for 3D CFD simulations, large computational resources are needed which limits their use in engineering practice
- For **wind farm analysis and optimization** of their power output, combine models (e.g. CFD & actuator disc model) are utilized to reduce computational costs

**In pursuit of the most appropriate modelling strategy, significant effort should be always spent on model validation**



**Thank you  
for your attention**

